Curie-Weiss model of the quantum measurement process

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A hamiltonian model is solved, which satisfies all requirements for a realistic ideal quantum measurement. The system S is a spin-$\frac{1}{2}$, whose z-component is measured through coupling with an apparatus A = M+B, consisting of a magnet M formed by a set of $N \gg 1$ spins with quartic infinite-range Ising interactions, and a phonon bath B at temperature T. Initially A is in a metastable paramagnetic phase. The process involves several time-scales. Without being much affected, A first acts on S, whose state collapses in a very brief time. The mechanism differs from the usual decoherence. Soon after its irreversibility is achieved. Finally the field induced by S on M, which may take two opposite values with probabilities given by Born's rule, drives A into its up or down ferromagnetic phase. The overall final state involves the expected correlations between the result registered in M and the state of S. The measurement is thus accounted for by standard quantum statistical mechanics and its specific features arise from the macroscopic size of the apparatus.

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The quantum measurement problem has given rise to an immense literature [1], but it is still an object of debate. Insight can be gained by studying exactly solvable dynamical models [2]. The one we consider hereafter is intended to be as realistic as possible.

We first have to face the irreducibly probabilistic nature of quantum mechanics, arising from the non-commutation of observables. Within the statistical interpretation, see e.g. [3], a wave-function can provide us with nothing more than the probabilities for the values of any physical quantity. We adhere to the so-called bayesian view on probabilities [4]: they are not inherent to a single object but refer to a population to which it belongs (in reality or in thought); they are mathematical tools for deducing sensible predictions from given prior knowledge. Thus, what is called a quantum "state", whether pure or mixed, gathers our information on the considered system: it does not characterize this system by itself, but its statistical ensemble. Accordingly, the consistency of quantum mechanics requires a measurement to be analyzed as a statistical process involving many similar experiments with all possible outcomes. This process couples a system (S) to an apparatus (A), creating correlations between the initial state (in the above sense) of S and the final value A_i of a pointer variable $\hat{A}$ of A. One infers thereby information about S as regards the occurrence of the corresponding eigenvalue s_i of one of its observables $\hat{s}$. This knowledge has statistical nature because quantum theory cannot inform us fully about a single object, even if it is "completely" prepared in a pure state.

Quantum statistical mechanics is also unavoidable for other reasons. On the one hand, the apparatus should be *macroscopic* so as to ensure registration of the outcomes A_i. The recorded results may later on be read by an observer whom we thus can leave aside. The required large size of A forbids us to assign a pure state to it since it cannot be completely prepared at the microscopic level. Rather, as always in statistical problems, *we must assign to it a density matrix.* On the other hand, a quantum measurement is an *irreversible* process. Statistical mechanics is necessary to explain this specific type of irreversibility, as any other one, by relying on microscopic *hamiltonian* dynamics.

Even if the coupling of S and A is weak, the perturbation it induces on S cannot in general be neglected in a quantum measurement. We will consider a so-called ideal measurement, which perturbs S as little as possible, keeping unchanged the statistics of all observables which commute with $\hat{s}$. The final density matrix of S, $r(t_{\rm f})$, is then obtained from $r(0)$ by cancelling the off-diagonal blocks associated with different eigenvalues s_i and s_j of $\hat{s}$. We denote the remaining diagonal blocks as r_i.

To represent a measurement, a process should have several specific features [1,5,6]. *i*) The apparatus A is macroscopic and at the initial time $t = 0$ in a metastable state $\mathcal{R}(0)$, independent of the arbitrary state $r(0)$ of the system S. The full density operator has the form

$$\mathcal{D}(0) = r(0) \otimes \mathcal{R}(0). \qquad (1)$$

ii) Triggered by its coupling with S, A may reach at the end of the process one among several possible states $\mathcal{R}_i$, which are a priori equally probable so as to avoid any bias. *iii*) Each state $\mathcal{R}_i$ is stable so as to register information robustly and permanently. The pointer variable $\hat{A}$ has in the state $\mathcal{R}_i$ negligible fluctuations around A_i so as to ensure precise and clear distinction between the possible outcomes. *iv*) The observable $\hat{s}$ of S does not change much during the process, and thus nearly commutes with the Hamiltonian. *v*) An ideal measurement involves a collapse, which changes $r(0)$ into the sum of its

diagonal blocks r_i corresponding to each s_i. *vi*) The density operator at the end of the process involves a special type of classical correlations between S and A, namely,

$$\mathcal{D}(0) \mapsto \mathcal{D}(t_{\rm f}) = \sum_i r_i \otimes \mathcal{R}_i \equiv \sum_i p_i \times \frac{r_i}{{\rm tr}\, r_i} \otimes \mathcal{R}_i. \quad (2)$$

Together with *iii* and with probabilistic interpretation of a quantum state, Eq. (2) means that the pointer variable may take any of the values A_i corresponding to s_i with the probabilities $p_i = {\rm tr}\, r_i$ (Born's law), and that if we select A_i, the subsequent statistics of S is described by the density operator $r_i/{\rm tr}\, r_i$ (von Neumann's reduction).

Most of these features have been emphasized in the past, and the models already worked out exhibit some among them [1,2,3,5,6]. In the present model *all* the above requirements will be fulfilled. In order to satisfy conditions *ii* and *iii* we take for A a macroscopic system displaying a phase transition with broken symmetry so as to eliminate any bias; $\hat{A}$ is an order parameter with small fluctuations. Moreover, A involves a large number of degrees of freedom which ensure an irreversible relaxation towards one of the equilibrium states $\mathcal{R}_i$.

The model. Our system S is a spin-$\frac{1}{2}$, the observable to be measured is its third Pauli matrix $\hat{s}_z$ with eigenvalues s_i equal to ± 1. Our apparatus A = M + B simulates a magnetic dot: M consists of $N \gg 1$ spins with Pauli operators $\hat{\sigma}_a^{(n)}$ $(a = x, y, z)$ coupled to the phonon bath B. The order parameter $\hat{A}$ is the magnetization in the z-direction $\hat{m} = \frac{1}{N}\sum_{n=1}^{N} \hat{\sigma}_z^{(n)}$. The Hamiltonian $\hat{H} = \hat{H}_{\rm A} + \hat{H}_{\rm SA}$ governing the measurement process reads:

$$\hat{H}_{\rm A} = \hat{H}_{\rm M} + \hat{H}_{\rm MB} + \hat{H}_{\rm B}, \quad \hat{H}_{\rm SA} = -gN\hat{m}\hat{s}_z, \quad (3)$$

$$\hat{H}_{\rm M} = -\frac{1}{q}\, JN\hat{m}^q, \quad \hat{H}_{\rm MB} = \sqrt{\gamma}\sum_{n=1}^{N}\sum_{a=x,y,z} \hat{\sigma}_a^{(n)}\hat{B}_a^{(n)}.$$

It commutes with $\hat{s}_z$ in agreement with [7], ensuring the condition *iv* above. The interaction $\hat{H}_{\rm SA}$ is a spin-spin coupling with strength $g > 0$. The Hamiltonian $\hat{H}_{\rm M}$, of the Curie-Weiss-type, couples all spins symmetrically. For $q = 2$ one has an Ising model, while $q = 4$ or 6 describes so-called super-exchange interactions as realized for metamagnets. We take $q = 4$. The bath B describes a set of Debye phonons at equilibrium in the thermodynamic limit. The dimensionless constant γ characterizes the strength of their coupling to the spins of M. It should be weak to trace out B and to ensure an exact Boltzmann-Gibbs distribution of M at equilibrium. The properties of the bath operators $\hat{B}_a^{(n)}$ will be specified later.

Mean-field approximation for the apparatus. The long-range character of $\hat{H}_{\rm M}$ ensures that its equilibrium behavior is exactly described in the large-N limit by the mean-field approximation. However, the correlations between S and A induced by $\hat{H}_{\rm SA}$ are essential for our purposes. Their very existence prevents us from applying the standard mean-field method. A way out of this is to separate the state $\mathcal{D}$ of the total system S + M + B into several sectors, and use a *different* time-dependent mean-field in each of them. In the eigenbasis of $\hat{s}_z$ for S, $|i\rangle = |\!\uparrow\rangle$ or $|\!\downarrow\rangle$ for eigenvalues $s_i = \pm 1$, $\mathcal{D}$ has the elements $\mathcal{D}_{ij} = \langle i|\mathcal{D}|j\rangle$. The von Neumann equation reads for each of the operators $\mathcal{D}_{ij}$ in the M + B space

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}_{ij} = -gN(s_i\hat{m}\mathcal{D}_{ij} - s_j\mathcal{D}_{ij}\hat{m}) + [\hat{H}_{\rm A}, \mathcal{D}_{ij}]. \quad (4)$$

The mean-field approach is implemented for each $\mathcal{D}_{ij}$. In $\hat{H}_{\rm A}$ we replace $\hat{m}^4$ by $m_{ij}^4 + 4m_{ij}^3(\hat{m} - m_{ij})$, where the c-number m_{ij} is determined selfconsistently. To do this we note that (unlike $\mathcal{D}_{\uparrow\uparrow}$ and $\mathcal{D}_{\downarrow\downarrow}$) $\mathcal{D}_{\uparrow\downarrow}$ and $\mathcal{D}_{\downarrow\uparrow} = \mathcal{D}_{\uparrow\downarrow}^\dagger$ are neither positive nor hermitian. Taking

$$m_{ij} = \frac{{\rm tr}\,\hat{m}\,|\mathcal{D}_{ij}|}{{\rm tr}\,|\mathcal{D}_{ij}|}, \qquad |\mathcal{D}_{ij}| \equiv \sqrt{\mathcal{D}_{ij}\mathcal{D}_{ij}^\dagger} \quad (5)$$

for any pair ij, one can show that this approximation becomes exact for large N, as in the static case.

Properties of the bath. The interaction between the bath B and the magnet M is treated within the cumulant weak-coupling approach, see e.g. [8]. The initial state of B is gibbsian at temperature $T = 1/\beta$, $R_{\rm B}(0) = \exp(-\beta\hat{H}_{\rm B})/Z_{\rm B}$, and is not correlated with the density operator $D(0) = r(0) \otimes R_{\rm M}(0)$ of S + M. The Hamiltonian $\hat{H}_{\rm B}$ is such that the free correlation functions of the bath variables are stationary, identical and independent for different components a and different sites n,

$${\rm tr}_{\rm B}[\, R_{\rm B}(0)\hat{B}_a^{(n)}(t)\hat{B}_b^{(m)}(s)\,] = \delta_{a,b}\,\delta_{n,m}\,K(t-s). \quad (6)$$

In agreement with the initial equilibrium of B, we assume a quasi-ohmic spectrum

$$K(t) = \hbar^2 \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{16\pi}\, \mathrm{e}^{\mathrm{i}\omega t}\,\omega\,(\coth\frac{1}{2}\beta\hbar\omega - 1)\,\mathrm{e}^{-|\omega|/\Gamma}, \quad (7)$$

with Γ the Debye frequency cut-off, where $\hbar\Gamma$ exceeds all other energy scales: T, J and g. The correlation time of B, of order $\hbar/T$, is much smaller than the characteristic times of A variables, which for long times ensures relaxation of A towards the Gibbs distribution. Besides the weak-coupling limit $\gamma \ll 1$, this requires not too low a temperature ($T \gg \gamma J$).

Bloch equations. As indicated above, in the large N limit each block of $\mathcal{D}$, and hence of $D = {\rm tr}_{\rm B}\,\mathcal{D}$, is of the time-dependent mean-field type:

$$D_{ij}(t) = r_{ij}(0) \times \rho_{ij}^{(1)}(t) \otimes \cdots \otimes \rho_{ij}^{(N)}(t). \quad (8)$$

Each $\rho_{ij}^{(n)}$ lives in the 2×2 Hilbert space of the n'th spin of M, and reads in the polarization representation: $\rho_{ij}^{(n)} = \frac{1}{2}\sum_{a=0,x,y,z}\zeta_{a,\,ij}\,\hat{\sigma}_a^{(n)}$, where the $\hat{\sigma}_0^{(n)}$ are 2×2

identity matrices and where $\zeta_{a,\,ij} = \mathrm{tr}\,\hat\sigma_a^{(n)}\rho_{ij}^{(n)}$ is independent of n.

As the coupling is weak, we can eliminate the bath from (4), even for times shorter than the memory-time of $K(t)$; Eq. (8) yields $\zeta_{x,\,ij} = \zeta_{y,\,ij} = 0$ at all t and

$$\dot\zeta_{0,\uparrow\downarrow} = \frac{2\mathrm{i}g}{\hbar}\zeta_{z,\uparrow\downarrow}, \quad \dot\zeta_{z,\uparrow\downarrow} = \frac{2\mathrm{i}g[1+\eta(t)]}{\hbar}\zeta_{0,\uparrow\downarrow} - 2\Lambda(t)\,\zeta_{z,\uparrow\downarrow}, \tag{9}$$

where for $t \ll 1/\Gamma$, Λ equals $\gamma\Gamma^2 t/2\pi$ and $\eta = \gamma\Gamma^2 t^2/2\pi$. They go in the markovian limit $t \gg \hbar/T$ to $\Lambda(\infty) = \gamma g/(2\hbar\tanh\beta g)$ and $\eta(\infty) = 0$.

For $\zeta_{0,\uparrow\uparrow}$ and for the magnetization $m_\uparrow = \zeta_{z,\uparrow\uparrow}/\zeta_{0,\uparrow\uparrow}$ we shall need below the only markovian equation:

$$\dot\zeta_{0,\uparrow\uparrow} = 0, \quad \dot m_\uparrow = \frac{\gamma h_\uparrow}{\hbar}\left(1 - \frac{m_\uparrow}{\tanh\beta h_\uparrow}\right), \tag{10}$$

where $h_\uparrow(t) = g + Jm_\uparrow^3(t)$ is the effective field in the considered sector.The sign of g is changed for the $\downarrow\downarrow$ sector.

Initial conditions. Before the measurement, M is prepared in a paramagnetic state $R_\mathrm{M}(0) = 2^{-N}\prod_n \hat\sigma_0^{(n)}$, leading to the initial density matrix of A = M + B: $\mathcal{R}(0) = R_\mathrm{M}(0)\otimes R_\mathrm{B}(0)$. The measurement will be unbiased, since the order parameter vanishes initially, $m_{ij}(0) = 0$. According to Eqs. (1, 8) we have $\zeta_{0,\,ij}(0) = 1$, $\zeta_{a,\,ij}(0) = 0$. The paramagnetic state is metastable for temperatures below $0.36J$ and this is the regime where A can act as a measuring apparatus. Its decay-time is then exponentially large in N, thus basically infinite. However, a change in the macroscopic state of the A can be induced by its coupling with S, provided g is sufficiently large so as to suppress the barrier leading to the lowest ferromagnetic state.

Collapse. We first consider the evolution of the off-diagonal elements $r_{\uparrow\downarrow}$, given by (8) as

$$r_{\uparrow\downarrow}(t) = \mathrm{tr}_{\mathrm{M,B}}\mathcal{D}_{\uparrow\downarrow}(t) = \mathrm{tr}_\mathrm{M} D_{\uparrow\downarrow}(t) = r_{\uparrow\downarrow}(0)\,\zeta_{0,\uparrow\downarrow}^N(t). \tag{11}$$

Obtained by solving (9) with its initial conditions by the exact WKB-method, $\zeta_{0,\,\uparrow\downarrow}(t)$ has the form $\exp[-\chi(t)]\cos\theta(t)$, where $\theta(t) = \frac{2g}{\hbar}\int_0^t \mathrm{d}s\,\exp 2[\chi(s) - \int_0^s \mathrm{d}u\,\Lambda(u)]$. For $t < 1/\Gamma$, χ and θ behave as

$$\chi \approx \frac{\gamma\Gamma^2 g^2}{2\pi\hbar^2}\,t^4, \quad \theta \approx \frac{2gt}{\hbar}\left(1 - \frac{\gamma\Gamma^2 t^2}{6\pi}\right). \tag{12}$$

(The markovian regime $t \gg \hbar/T$, $\chi = \Lambda(\infty)t$ is irrelevant here when N is very large.) The amplitude of $\zeta_{0,\uparrow\downarrow}$ decreases as an exponential, quartic in t for small times and linear for long times. Since $\zeta_{z,\uparrow\downarrow}$ is imaginary, $\mathcal{D}_{\uparrow\downarrow}\mathcal{D}_{\uparrow\downarrow}^\dagger$ is proportional to the unit matrix, so that $m_{\uparrow\downarrow}$ vanishes.

We thus find for $r_{\uparrow\downarrow}(t) = r_{\uparrow\downarrow}(0)\mathrm{e}^{-N\chi(t)}[\cos\theta(t)]^N$ a sequence of narrow gaussian peaks, arising from the nearly periodic cosine factor and located around the times at which θ/π is an integer. At the very beginning of the measurement, (12) show that the off-diagonal elements $r_{\uparrow\downarrow}(t) = r_{\downarrow\uparrow}^*(t)$ of the marginal density matrix of S rapidly fall down. For a large apparatus such that $N \gg \gamma\,(\hbar\Gamma/g)^2$, this decrease is dominated by that of $[\cos(2gt/\hbar)]^N = \exp[-(t/\tau_\mathrm{collapse})^2]$ rather than that of $\exp[-N\chi]$. The collapse time

$$\tau_\mathrm{collapse} = \frac{1}{\sqrt{2N}}\,\frac{\hbar}{g}, \tag{13}$$

thus characterizes the disappearance of the components $D_{\uparrow\downarrow}$ and $D_{\downarrow\uparrow}$ for S+M. This time is much shorter than all other characteristic times of the process. Its coefficient $\hbar/g$ differs from the one $\hbar/T$ that enters standard decoherence times.

The subsequent spikes, the first of which occurs at $t = \pi\hbar/(2g)$, are suppressed by the factor $\exp[-N\chi(t)]$ which arises due to the interaction with the bath, a decay amplified by the large value of N. Due to (12) this means that after a decoherence time

$$\tau_\mathrm{decoh} = \left(\frac{2\pi}{\gamma N}\right)^{1/4}\left(\frac{\hbar}{\Gamma g}\right)^{1/2} \tag{14}$$

all the peaks are washed out, and $r_{\uparrow\downarrow}$ and $r_{\downarrow\uparrow}$ remain zero after their initial gaussian collapse, while $r_{\uparrow\uparrow}$ and $r_{\downarrow\downarrow}$ are kept unchanged since $\hat s_z$ is conserved. The collapse proper thus results only from the interaction $\hat H_\mathrm{SA}$ of S with the large number of spins, as shown by (13). The irreversible loss of information about the off-diagonal elements takes place at a characteristic time τ_decoh, after the collapse. Provided $N \gg (\hbar\Gamma/g)^2/\gamma$, it is faster than $1/\Gamma$, the attempt time of the bath, and well before the recurrence of the first peak, since we assumed already that $\hbar\Gamma \gg g$. This irreversibility is essential although hidden, since $r_{\uparrow\downarrow}$ has not yet revived when it takes place.

In fact, not only the initial collapse, but even the suppression of possible revivals, do not require M to interact with a bath. A realistic interaction $\hat H_\mathrm{SA}$ would involve a small dispersion δg around the average value $\langle g\rangle$ of the coupling between $\hat s_z$ and the various spins $\hat\sigma_z^{(n)}$ of A. For independent disorder a cumulant expansion now brings $\theta = \langle g\rangle 2t/\hbar - \langle\delta g^3\rangle_\mathrm{c}(2t/\hbar)^3/3! + \cdots$ and $\chi = \langle\delta g^2\rangle_\mathrm{c}(2t/\hbar)^2/2 - \langle\delta g^4\rangle_\mathrm{c}(2t/\hbar)^4/4! + \cdots$. The damping factor $\exp[-N\chi(t)]$ then suppresses the recurrent peaks provided $N \gg \langle g\rangle^2/\langle\delta g^2\rangle_\mathrm{c}$. The large size of the apparatus thus suffices to make the initial collapse permanent. The irreversibility of the collapse is here as a collective effect due to the large value of N and not to the environment. These two mechanisms for suppression of the x, y components of the spin S are reminiscent of the spin-lattice and spin-spin relaxation mechanisms in NMR, respectively.

Registration. Let us now consider the evolution of the diagonal elements $D_{\uparrow\uparrow}$ and $D_{\downarrow\downarrow}$. Since they are not affected by the initial process, their evolution is governed

by (10) in the markovian regime $t \gg \hbar/T$. The registration by the apparatus will therefore look like a relaxation towards equilibrium in statistical mechanics. The Curie-Weiss equation, $m_i = \tanh[\beta h_i]$ with $h_i = \pm g + J m_i^3$ for each sector $i = \uparrow$ of $\downarrow$, gives the extrema of the free energy per site $F_i(m) = \mp gm - \frac{1}{4}Jm^4 - TS(m)$, where $S(m) = -\frac{1+m}{2}\ln\frac{1+m}{2} - \frac{1-m}{2}\ln\frac{1-m}{2}$ is entropy in the mean-field approximation. The minima of $F_i(m)$ are attractors for the evolution (10). Since $m_\uparrow$ begins to increase as $\gamma g t/\hbar$, it relaxes to the smallest positive value of m where $F_\uparrow(m)$ is minimal. If the temperature is not sufficiently low, or if g is too small, this is the paramagnetic state, $m_\uparrow \to \tanh\beta g$ and $m_\downarrow \to -\tanh\beta g$. The result of the measurement cannot then be registrated robustly, since, after the coupling with S is removed, M returns to $m_\uparrow = m_\downarrow = 0$. However, for sufficiently large g ($g > 0.08J$ for $T = 0.34J$), the paramagnetic state is totally lost and the relaxation of M leads to a ferromagnetic state with magnetization nearly equal to 1 for $m_\uparrow$, to -1 for $m_\downarrow$ (± 0.996 for $T = 0.34J$, $g = 0.09J$). When the coupling is switched off after relaxation, M remains in the vicinity of that state: the system A=M+B is non-ergodic and the memory of its triggering by S is kept forever. The duration of the measurement, namely

$$\tau_{\rm meas} = \frac{\hbar}{\gamma g}. \qquad (15)$$

is governed by the establishment of strong correlations of M and S, which takes place when the magnetization of M reaches significant values having the same sign as s_z. This stage of the process is the slowest of all. The dimensionless factor $1/\gamma$ expresses that relaxation occurs due to coupling with the bath B. The final stage, after $m_\uparrow$ has become sizeable, is a more rapid exponential relaxation with characteristic time $\hbar/\gamma J$, during which the coupling with S is ineffective, and which leads to robust registration in a ferromagnetic state.

Altogether, the common final state of A and S after $\tau_{\rm meas}$ has the form (2) with probabilities $p_\uparrow = r_{\uparrow\uparrow}(0)$, $p_\downarrow = r_{\downarrow\downarrow}(0)$ for $i = \uparrow$ or $\downarrow$; the states $|\uparrow\rangle\langle\uparrow|$ or $|\downarrow\rangle\langle\downarrow|$ of S are correlated with the ferromagnetic states of A, $\mathcal{R}_\uparrow$ or $\mathcal{R}_\downarrow$, having positive or negative magnetization, respectively.

Conclusion. In spite of the simplicity of the present model, its exact solution displays all the features, listed in the introduction, that a quantum measurement should satisfy. We relied on the statistical interpretation of quantum mechanics, which naturally leads to describe an ensemble of measurements on an ensemble of systems and which should yield all possible outcomes with Born probabilities. The process follows an elaborate scenario involving *several time-scales*. At the very beginning, over the very short time (13), the state of S collapses, while A is affected only microscopically. This collapse is governed by the large value of N and should be contrasted with the standard decoherence processes [1,2]. Somewhat later, it is made irreversible, either by means of the interaction with the thermal bath (at the decoherence time (14)), or under the effect of a small randomness in the coupling of S with M. This stage is invisible, since it corresponds only to a disappearance of complicated many-spin correlations within M; its duration is longer than $\tau_{\rm collapse}$ but shorter than the recurrence time $\pi\hbar/(2g)$ which would exist without any dissipation. Thereafter the statistics of S becomes classical for the two values $s_z = \pm 1$ and remains unchanged in time. The system S, although microscopic, is seen by M as an external magnetic field $\pm g$ which is sufficient to trigger the subsequent evolution of M from its initial metastable state towards either one of its ferromagnetic states. Because M is macroscopic, this evolution is slow; its time-scale $\tau_{\rm meas}$ is governed by the bath B (the parameters of which satisfy $\hbar\Gamma \gg T \gg \gamma J$ and $\hbar\Gamma \gg J > g$). Finally, the registration becomes permanent owing to the irreversible relaxation of M into stable equilibrium. This takes place over a time of order $\hbar/(\gamma J)$ shorter than the time $\hbar/(\gamma g)$ required to leave the metastable state. We have focused on parameters for which the model simulates an ideal measurement, but the analysis may be extended to cover realistic imperfect measurements, for instance if N is not very large or if the observed quantity $\hat{s}_z$ is not conserved.

An essential property allowing the process to be used as an ideal measurement is the *macroscopic size of the apparatus*. It is the large value of N and the quasi-continuity of the phonon spectrum which ensure the collapse, the breaking of invariance of M which generates its initial metastable state and its final two possible stable states, and the dynamics of A which leads to registration. The macroscopic nature of A thus conditions the form (2) of the outcoming state, with its correlations of classical nature between the sign of the magnetization of M and the final marginal state $|\uparrow\rangle\langle\uparrow|$ or $|\downarrow\rangle\langle\downarrow|$ of S, and with the occurrence of the Born probabilities $p_\uparrow$ or $p_\downarrow$ for each of these events. This statistical description of the final state expresses that each particular experiment yields a well-defined outcome for M, and that the selection of a given result for M can serve as a preparation of S immediately after the process in one of its pure states $|\uparrow\rangle$ or $|\downarrow\rangle$. The *emergence of classical probabilities from quantum mechanics* is thus a macroscopic phenomenon, explained by means of quantum statistical mechanics and occurring on definite time-scales. It is comparable with macroscopic irreversibility, which emerges from hamiltonian dynamics for many degrees of freedom in the framework of statistical mechanics. Actually, here also, several mechanisms come into play, which explain the irreversibility of the measurement process; but moreover they transform the microscopic non-commutative probabilistic description of quantum mechanics into ordinary probabilities for the final state.

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